Published by Institute of Physics Publishing for SISSA/ISAS

Received: *July 22, 2003*
Revised: *September 26, 2003*
Accepted: *September 30, 2003*

Physics impact of a precise determination of the top quark mass at an e^+e^- linear collider

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Abstract: At a prospective e^+e^- Linear Collider (LC) a very precise determination of the top quark mass with an accuracy of $\delta m_t \lesssim 100\,\mathrm{MeV}$ will be possible. This is to be compared with the envisaged accuracy of $\delta m_t = 1$–$2\,\mathrm{Gev}$ at the Tevatron and the LHC. We discuss the physics impact of such a precise determination of m_t, focusing on the Standard Model (SM) and its minimal supersymmetric extension (MSSM). In particular, we show the importance of a precise knowledge of m_t for electroweak precision observables, and for Higgs physics and the scalar top sector of the MSSM. Taking the mSUGRA model as a specific example, we furthermore demonstrate the importance of a precise m_t value for the prediction of sparticle masses and for constraints on the parameter space allowed by the relic density. The uncertainty in m_t also significantly affects the reconstruction of the supersymmetric high scale theory. We find that going from hadron collider to LC accuracy in m_t leads to an improvement of the investigated quantities by up to an order of magnitude.

Keywords: Standard Model, Quark Masses and SM Parameters, Supersymmetric Standard Model, Higgs Physics.

Contents

1. Introduction

At a prospective e^+e^- Linear Collider (LC) a very precise determination of the top quark mass with an accuracy of

$$\delta m_t \lesssim 100\,\text{MeV} \quad (\text{LC}) \tag{1.1}$$

will be possible [1]–[6]. This has to be compared with the envisaged accuracy of

$$\delta m_t = 1\text{–}2\,\text{GeV} \quad (\text{Tevatron, LHC}) \tag{1.2}$$

at hadron colliders, i.e. Run II of the Tevatron and the LHC [7]. The question arises of what can be learnt from the improved accuracy obtainable at the LC. Some implications have recently been discussed in ref. [8]. In the present paper, we perform a detailed investigation of the physics impact of the LC precision on m_t, focusing on the Standard Model (SM) and the Minimal Supersymmetric Standard Model (MSSM). We study the dependence of different quantities on m_t and compare these effects with the anticipated future experimental accuracies, taking into account theoretical uncertainties induced by the experimental errors of other input parameters, as well as uncertainties from unknown higher-order corrections.

We discuss in particular possible improvements in the analysis of electroweak precision observables induced by a precision measurement of m_t. Moreover, we demonstrate the impact of the achievable precision in m_t on the phenomenology of the Higgs and stop sectors of the MSSM, on the possibility to reconstruct the underlying high scale theory, and on constraints from the dark matter relic density.

While the examples presented in this paper are by no means exclusive, we believe that they give a fair idea of the physics impact of a very precise determination of m_t. Other examples where m_t is an important parameter are, for example, B and K physics. B and K decays and B^0–$\bar{B}^0$ mixing are, however, significantly affected by hadronic uncertainties, such that a m_t precision of a few GeV is sufficient; see also ref. [8]. Also for the high-precision determination of $\alpha_s(M_Z)$ at the Z-boson resonance (GigaZ) it turned out that $\delta m_t \sim 1\,\mathrm{GeV}$ will be sufficient [9].

In our analysis we do not discuss different definitions of the top quark mass. The accuracy of the mass parameter extracted from the $t\bar{t}$ threshold measurements at the LC will be significantly better than the 100 MeV quoted in eq. (1.1), see [1]–[6]. However, its transition into a short-distance mass (like the $\overline{\mathrm{MS}}$ mass) that is suitable as an input parameter for the observables that we investigate below, involves further theoretical uncertainties. Taking these uncertainties into account, an accuracy in the m_t determination of $\delta m_t \lesssim 100\,\mathrm{MeV}$ as quoted in eq. (1.1) seems to be feasible (for a review, see [4] and references therein). This requires an experimental accuracy of $\alpha_s(M_Z)$ of about 0.001, which can be obtained from LC measurements also at the $t\bar{t}$ threshold [5], possibly from event shape measurements at the LC [6], and from LC measurements at GigaZ, where an even higher accuracy seems to be achievable [9].

The remaining parts of the paper are organized as follows: section 2 focuses on the electroweak precision observables in the SM and the MSSM. Internal consistency checks of the two models are investigated, and the impact of the precision in m_t is analysed with respect to the experimental accuracy of the precision observables and the theoretical uncertainties of the predictions. The influence of δm_t on precision physics in the MSSM Higgs and stop sectors is analysed in section 3. In section 4 we study the relevance of δm_t in renormalization group (RG) running of SUSY parameters. We investigate the consequences for chargino and neutralino mass predictions, dark matter constraints on mSUGRA parameters, and for the reconstruction of the SUSY–breaking boundary conditions. Section 5 gives a summary and conclusions.

2. Electroweak precision observables

2.1 Consistency tests of the SM and the MSSM

Electroweak precision observables (EWPO) can be used to perform internal consistency checks of the model under consideration and to obtain indirect constraints on the unknown model parameters. This is done by comparing the experimental results of the EWPO with their theory prediction within, for example, the SM or the MSSM. In this subsection, we focus on the two most prominent observables in the context of electroweak precision tests, the W boson mass M_W and the effective leptonic mixing angle $\sin^2\theta_{\mathrm{eff}}$.

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Currently the uncertainty in m_t is by far the dominant effect in the theoretical uncertainties of the EWPO. Today's experimental errors of M_W and $\sin^2\theta_{\rm eff}$ [10] are shown in table 1, together with the prospective future experimental errors obtainable at the Tevatron [11], the LHC [12], and the LC without [1, 2, 13] and with a GigaZ option [14, 15] (see [16] for a compilation of these errors and additional references).

There are two sources of theoretical uncertainties: (i) uncertainties those from unknown higher-order corrections, which we call intrinsic theoretical uncertainties, and (ii) those from experimental errors of the input parameters, which we call parametric theoretical uncertainties. The intrinsic uncertainties within the SM are currently [16, 17]

$$\Delta M_W^{\rm intr,today} \approx \pm 4\,\mathrm{MeV}\,, \qquad \Delta\sin^2\theta_{\rm eff}^{\rm intr,today} \approx \pm 6\times 10^{-5}\,. \tag{2.1}$$

They are based on the present status of the theoretical predictions in the SM, namely the complete two-loop result for M_W (see [18] and references therein), a partial two-loop result for $\sin^2\theta_{\rm eff}$ (see [19] and references therein) and leading three-loop contributions to both observables (see [20] for the latest result, and references therein). For the MSSM, the available results beyond one-loop order are less advanced than in the SM (for the latest two-loop results, see [21] and references therein). Thus, the intrinsic uncertainties in the MSSM are still considerably larger than the ones quoted for the SM in eq. (2.1) [22].

The parametric uncertainties induced by the experimental errors of m_t and $\Delta\alpha_{\rm had}$ are currently [23]

$$\begin{aligned} \delta m_t &= 5.1\,\mathrm{GeV} \Rightarrow \Delta M_W^{{\rm para},m_t} \approx \pm 31\,\mathrm{MeV}\,, & \Delta\sin^2\theta_{\rm eff}^{{\rm para},m_t} &\approx \pm 16\times 10^{-5} \\ \delta(\Delta\alpha_{\rm had}) &= 36\times 10^{-5} \Rightarrow \Delta M_W^{{\rm para},\Delta\alpha_{\rm had}} \approx \pm 6.5\,\mathrm{MeV}\,, & \Delta\sin^2\theta_{\rm eff}^{{\rm para},\Delta\alpha_{\rm had}} &\approx \pm 13\times 10^{-5}\,. \end{aligned} \tag{2.2}$$

Accordingly, the parametric uncertainties of M_W and $\sin^2\theta_{\rm eff}$ induced by δm_t are approximately as large as the current experimental errors. For the case of M_W the parametric uncertainty $\Delta M_W^{{\rm para},m_t}$ is more than four times larger than $\Delta M_W^{{\rm para},\Delta\alpha_{\rm had}}$ and more than 15 times larger than $\Delta M_W^{\rm intr,today}$.

In order to exploit the high experimental precision of the EWPO obtainable at the next generation of colliders for constraining effects of new physics, a precise measurement of m_t is mandatory. The parametric uncertainties induced by future m_t measurements in comparison with the ones from a future error of $\Delta\alpha_{\rm had}$ [24] and from the error of M_Z are [7, 22]

$$\begin{aligned} \delta m_t &= 2\,\mathrm{GeV} \Rightarrow \Delta M_W^{{\rm para},m_t} \approx \pm 12\,\mathrm{MeV}\,, & \Delta\sin^2\theta_{\rm eff}^{{\rm para},m_t} &\approx \pm 6\times 10^{-5} \\ \delta m_t &= 1\,\mathrm{GeV} \Rightarrow \Delta M_W^{{\rm para},m_t} \approx \pm 6\,\mathrm{MeV}\,, & \Delta\sin^2\theta_{\rm eff}^{{\rm para},m_t} &\approx \pm 3\times 10^{-5} \\ \delta m_t &= 0.1\,\mathrm{GeV} \Rightarrow \Delta M_W^{{\rm para},m_t} \approx \pm 1\,\mathrm{MeV}\,, & \Delta\sin^2\theta_{\rm eff}^{{\rm para},m_t} &\approx \pm 0.3\times 10^{-5} \\ \delta(\Delta\alpha_{\rm had}) &= 5\times 10^{-5} \Rightarrow \Delta M_W^{{\rm para},\Delta\alpha_{\rm had}} \approx \pm 1\,\mathrm{MeV}\,, & \Delta\sin^2\theta_{\rm eff}^{{\rm para},\Delta\alpha_{\rm had}} &\approx \pm 1.8\times 10^{-5} \\ \delta M_Z &= 2.1\,\mathrm{MeV} \Rightarrow \Delta M_W^{{\rm para},M_Z} \approx \pm 2.5\,\mathrm{MeV}\,, & \Delta\sin^2\theta_{\rm eff}^{{\rm para},M_Z} &\approx \pm 1.4\times 10^{-5}\,. \end{aligned} \tag{2.3}$$

In order to keep the theoretical uncertainty induced by m_t at a level comparable to or smaller than the other parametric and intrinsic uncertainties (the latter are expected to further improve in the near future), δm_t has to be smaller than about 0.2 GeV in the case

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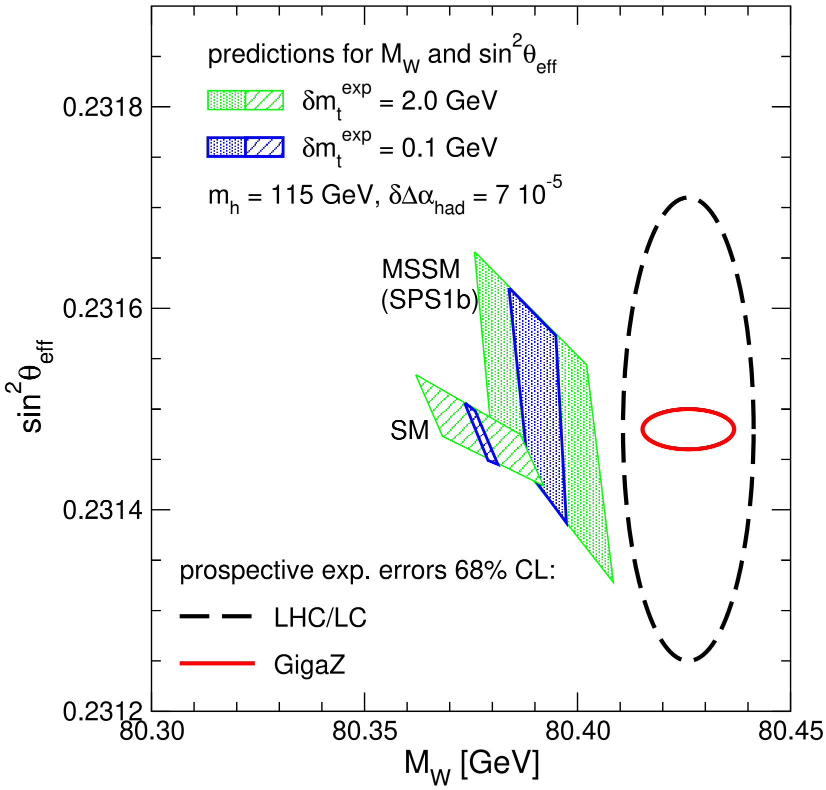

Figure 1: The predictions for M_W and $\sin^2\theta_{\text{eff}}$ in the SM and the MSSM (SPS1b). The inner (blue) areas correspond to $\delta m_t = 0.1\,\text{GeV}$ (LC), while the outer (green) areas arise from $\delta m_t = 2\,\text{GeV}$ (LHC). The anticipated experimental errors on M_W and $\sin^2\theta_{\text{eff}}$ at the LHC/LC and at a LC with GigaZ option are indicated.

	Today	Tevatron/LHC	LC	GigaZ
$\delta\sin^2\theta_{\text{eff}}(\times 10^5)$	17	14–20	–	1.3
δM_W [MeV]	34	15	10	7

Table 1: Experimental errors of M_W and $\sin^2\theta_{\text{eff}}$ at present and future colliders [1, 2] and [10]–[16].

of M_W, and about $0.5\,\text{GeV}$ in the case of $\sin^2\theta_{\text{eff}}$.[1] This level of theoretical accuracy is necessary in order to exploit the prospective experimental precision of the EWPO at a LC with GigaZ option, see table 1.

As an example for the potential of a precise measurement of the EWPO to explore the effects of new physics, we show in figure 1 the predictions for M_W and $\sin^2\theta_{\text{eff}}$ in the SM and the MSSM in comparison with the prospective experimental accuracy obtainable at the LHC and an LC without GigaZ option (labelled as LHC/LC) and with the accuracy obtainable at an LC with GigaZ option (labelled as GigaZ). The current experimental values

[1]The parametric theoretical uncertainty of $\sin^2\theta_{\text{eff}}$ appears to be limited by the prospective accuracy of $\Delta\alpha_{\text{had}}$ in eq. (2.3). However, the theoretical uncertainty of $\sin^2\theta_{\text{eff}}$ can be further reduced in this case by trading $\Delta\alpha_{\text{had}}$ as an input parameter for the Fermi constant G_F and M_W, see eq. (3.4) below, leading in this way to an even stricter requirement on the experimental precision of m_t.

are taken as the central ones [10]. For the Higgs boson mass a future measured value of $m_h = 115\,\mathrm{GeV}$ has been assumed (in accordance with the final lower bound obtained at LEP [25]). The MSSM parameters have been chosen in this example according to the reference point SPS1b [26]. In figure 1 the inner (blue) areas correspond to $\delta m_t = 0.1\,\mathrm{GeV}$ (LC), while the outer (green) areas arise from $\delta m_t = 2\,\mathrm{GeV}$ (LHC). For the error of $\Delta\alpha_{\mathrm{had}}$ we have again assumed a future determination of 5×10^{-5}. In the SM, this is the only relevant uncertainty apart from δm_t (the remaining effects of future intrinsic uncertainties have been neglected in this figure). The future experimental uncertainty of m_h is insignificant for electroweak precision tests. For the experimental errors on the SUSY parameters we have assumed a 5% uncertainty for $m_{\tilde{t}_1}, m_{\tilde{t}_2}, m_{\tilde{b}_1}, m_{\tilde{b}_2}$ around their values given by SPS1b. The mixing angles in the $\tilde{t}$ and $\tilde{b}$ sectors have been left unconstrained. The mass of the $\mathcal{CP}$-odd Higgs boson M_A is assumed to be determined to about 10%, and it is assumed that $\tan\beta \approx 30 \pm 4.5$, where $\tan\beta$ is the ratio of the vacuum expectation values of the two Higgs doublets of the MSSM.

The figure shows that the improvement in δm_t from $\delta m_t = 2\,\mathrm{GeV}$ to $\delta m_t = 0.1\,\mathrm{GeV}$ strongly reduces the parametric uncertainty in the prediction for the EWPO. In the SM case it leads to a reduction by about a factor of 10 in the allowed parameter space of the $M_W - \sin^2\theta_{\mathrm{eff}}$ plane. In the MSSM case, where many additional parametric uncertainties enter, a reduction by a factor of more than 2 is obtained in this example. This precision will be crucial to establish effects of new physics via EWPO.

2.2 Indirect determination of the SM top Yukawa coupling

A high precision on m_t is also important to obtain indirect constraints on the top Yukawa coupling y_t from EWPO [21]. The top Yukawa coupling enters the SM prediction of EWPO starting at $\mathcal{O}(\alpha\alpha_t)$ [27]. Indirect bounds on this coupling can be obtained if one assumes that the usual relation between the Yukawa coupling and the top quark mass, $y_t = \sqrt{2}\,m_t/v$ (where v is the vacuum expectation value), is modified.

Assuming a precision of $\delta m_t = 2\,\mathrm{GeV}$, an indirect determination of y_t with an accuracy of only about 80% can be obtained from the EWPO measured at an LC with GigaZ option. A precision of $\delta m_t = 0.1\,\mathrm{GeV}$, on the other hand, leads to an accuracy of the indirect determination of y_t of about 40%. The inclusion of further subleading terms beyond $\mathcal{O}(\alpha\alpha_t)$ would increase this precision; see the discussion in [21]. The indirect determination of y_t from EWPO is competitive with the indirect constraints from the $t\bar{t}$ threshold [5]. These indirect determinations of y_t represent an independent and complementary approach to the direct measurement of y_t via $t\bar{t}H$ production at the LC, which of course provides the highest accuracy [1, 28].

3. Implications for the MSSM

3.1 Radiative corrections in the MSSM Higgs boson sector

In contrast to the SM, where the Higgs boson mass is a free input parameter, the mass of the lightest $\mathcal{CP}$-even Higgs boson in the MSSM can be predicted in terms of other parameters of the model. Thus, precision measurements in the Higgs sector of the MSSM have the potential to play a similar role as the "conventional" EWPO for constraining the parameter space of the model and possible effects of new physics.

At the tree level, the masses of the neutral $\mathcal{CP}$-even Higgs bosons can be expressed in terms of M_Z, M_A and $\tan\beta = v_2/v_1$ as follows:

$$\begin{aligned} m^2_{h,\text{tree}} &= \frac{1}{2}\left[M_A^2 + M_Z^2 - \sqrt{(M_A^2+M_Z^2)^2 - 4M_Z^2M_A^2\cos^2 2\beta}\right] \\ m^2_{H,\text{tree}} &= \frac{1}{2}\left[M_A^2 + M_Z^2 + \sqrt{(M_A^2+M_Z^2)^2 - 4M_Z^2M_A^2\cos^2 2\beta}\right]. \end{aligned} \tag{3.1}$$

This implies an upper bound of $m_{h,\text{tree}} < M_Z$. The existence of such a bound, which does not occur in the case of the SM Higgs boson, can be related to the fact that the quartic term in the Higgs potential of the MSSM is given in terms of the gauge couplings, while the quartic coupling is a free parameter in the SM.

The tree-level bound, being obtained from the gauge couplings, receives large corrections from SUSY-breaking effects in the Yukawa sector of the theory. The leading one-loop correction is proportional to m_t^4. For instance, the leading logarithmic one-loop term (for vanishing mixing between the squarks) reads [29]

$$\Delta m_h^2 = \frac{3G_F m_t^4}{\sqrt{2}\,\pi^2\sin^2\beta}\ln\left(\frac{m_{\tilde{t}_1}m_{\tilde{t}_2}}{m_t^2}\right). \tag{3.2}$$

Corrections of this kind have dramatic effects on the predicted value of m_h and many other observables in the MSSM Higgs sector. The one-loop corrections can shift m_h by 50–100%. Since this shift is related to effects from a part of the theory that does not enter at tree level, corrections even of this size do not invalidate the perturbative treatment. In fact, the relative size of the one- and two-loop corrections [30]–[39] is in accordance with the expectation from a perturbative expansion.

Since these very large corrections are proportional to the fourth power of the top quark mass, the predictions for m_h and many other observables in the MSSM Higgs sector strongly depend on the value of m_t. As a rule of thumb [40], a shift of $\delta m_t = 1\,\text{GeV}$ induces a parametric theoretical uncertainty of m_h of also about $1\,\text{GeV}$, i.e. $\Delta m_h^{\delta m_t} \approx \delta m_t$.

Comparing a precise measurement of m_h and other Higgs sector observables with the MSSM prediction will allow us to obtain sensitive constraints on the MSSM parameters, in particular M_A, $\tan\beta$, and the parameters of the stop and sbottom sectors. A precise knowledge of m_t will clearly be mandatory in order to sensitively probe the MSSM parameters or even effects of physics beyond the MSSM.

In order to discuss the impact of a precise measurement of m_t on the phenomenology of the MSSM Higgs sector more quantitatively, we restrict our analysis to the lightest MSSM Higgs boson mass m_h for simplicity. Concerning the relevance of the experimental precision of m_t, as discussed above, three other sources of uncertainties have to be investigated, namely the experimental error, δm_h^{exp}, the parametric theoretical uncertainty from other SM input parameters, Δm_h^{para}, and the intrinsic theoretical uncertainty from unknown higher-order corrections, Δm_h^{intr}:

- Experimental error:
The prospective accuracies that can be obtained in the experimental determination of m_h at the LHC [41] and at the LC [1, 2] are:

$$\begin{aligned} \delta m_h^{\rm exp} &\approx 200\,\mathrm{MeV} \quad (\mathrm{LHC}) \\ \delta m_h^{\rm exp} &\approx 50\,\mathrm{MeV} \quad (\mathrm{LC})\,. \end{aligned} \tag{3.3}$$

- Parametric theoretical uncertainty from other SM input parameters:
Besides m_t, the other SM input parameters whose experimental errors can be relevant to the prediction of m_h are M_W, α_s, and m_b. The W boson mass M_W mainly enters via the reparameterization of the electromagnetic coupling α in terms of the Fermi constant G_F:

$$\alpha = \frac{\sqrt{2}\,G_F}{\pi} M_W^2 \left(1 - \frac{M_W^2}{M_Z^2}\right) \frac{1}{1+\Delta r}\,, \tag{3.4}$$

where the quantity Δr summarizes the radiative corrections.

The present experimental error of $\delta M_W^{\rm exp} = 34\,\mathrm{MeV}$ leads to a parametric theoretical uncertainty of m_h below $0.1\,\mathrm{GeV}$. In view of the prospective improvements in the experimental accuracy of M_W, the parametric uncertainty induced by M_W will be smaller than the one induced by m_t, even for $\delta m_t = 0.1\,\mathrm{GeV}$.

The current experimental error of the strong coupling constant, $\delta\alpha_s(M_Z) = 0.002$ [42], induces a parametric theoretical uncertainty of m_h of about $0.3\,\mathrm{GeV}$. Since a future improvement of the error of $\alpha_s(M_Z)$ by about a factor of 2 can be envisaged [9, 14, 42], the parametric uncertainty induced by m_t will dominate over the one induced by $\alpha_s(M_Z)$ down to the level of $\delta m_t = 0.1$–$0.2\,\mathrm{GeV}$.

The mass of the bottom quark currently has an experimental error of about $\delta m_b = 0.1\,\mathrm{GeV}$ [42, 43]. A future improvement of this error by about a factor of 2 seems to be feasible [43]. The influence of the bottom and sbottom loops on m_h depends on the parameter region, in particular on the values of $\tan\beta$ and μ (the higgsino parameter). For small $\tan\beta$ and/or μ the contribution from bottom and sbottom loops to m_h is typically below $1\,\mathrm{GeV}$, in which case the uncertainty induced by the current experimental error on m_b is completely negligible. For large values of $\tan\beta$ and μ, the effect of bottom/sbottom loops can exceed $10\,\mathrm{GeV}$ in m_h [38, 44]. Even in these cases we find that the uncertainty in m_h induced by $\delta m_b = 0.1\,\mathrm{GeV}$ rarely exceeds the level of $0.1\,\mathrm{GeV}$, since higher-order QCD corrections effectively reduce the bottom quark contributions. Thus, the parametric uncertainty induced by m_t will in general dominate over the one induced by m_b, even for $\delta m_t \approx 0.1\,\mathrm{GeV}$.

The comparison of the parametric uncertainties of m_h induced by the experimental errors of M_W, $\alpha_s(M_Z)$ and m_b with the one induced by the experimental error of the top quark mass shows that an uncertainty of $\delta m_t \approx 1\,\mathrm{GeV}$, corresponding to the accuracy achievable at the LHC, will be the dominant parametric uncertainty of m_h.

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The accuracy of $\delta m_t \approx 0.1\,\mathrm{GeV}$ achievable at the LC, on the other hand, will allow a reduction of the parametric theoretical uncertainty induced by δm_t to about the same level as the uncertainty induced by the other SM input parameters.

- Intrinsic error:
  Concerning the intrinsic theoretical uncertainty in the prediction for m_h from unknown higher-order corrections, considerable progress has been made over the last years. The full one-loop corrections [30], the leading corrections at $\mathcal{O}(\alpha_t\alpha_s)$ [31]–[34], the subleading $\mathcal{O}(\alpha_t^2)$ contributions [35, 36], as well as the leading $\mathcal{O}(\alpha_b\alpha_s, \alpha_t\alpha_b, \alpha_b^2)$ corrections [38] are available. Recently, also the full electroweak two-loop corrections in the approximation of vanishing external momentum have been published [39].

  However, at the present level of sophistication in the evaluation of two-loop contributions to m_h, the intrinsic uncertainty from unknown higher-order corrections is still estimated to be rather large [44, 45]:

$$\Delta m_h^{\mathrm{intr,today}} \approx 3\,\mathrm{GeV}\,. \tag{3.5}$$

  If this intrinsic uncertainty cannot drastically be reduced, it will clearly be the dominant theoretical uncertainty in the future. On the other hand, there are no principle obstacles that would prevent an improvement of the accuracy of the perturbative evaluation to the level of $\Delta m_h^{\mathrm{intr,future}} \approx 0.1\,\mathrm{GeV}$. This very ambitious goal clearly demands an enormous effort, requiring the bulk part of three-loop corrections and leading higher-order contributions. It does not seem to be out of reach, however, on the time scale of about a decade.

In figures 2–4, we focus on the experimental error of m_h and its parametric uncertainty, while the possible impact of the future intrinsic uncertainty is discussed in the text.

The relevance of the parametric uncertainty in m_h induced by different experimental errors on m_t is illustrated in figure 2, where the prediction for m_h is shown as a function of M_A in the $m_h^{\max}$ benchmark scenario [46]. The evaluation of m_h has been done with *FeynHiggs* [47] for a central value of the top quark mass of $m_t = 175\,\mathrm{GeV}$ and for $\tan\beta = 5$. The figure shows that a reduction of the experimental error from $\delta m_t = 1$–$2\,\mathrm{GeV}$ (LHC) to $\delta m_t = 0.1\,\mathrm{GeV}$ (LC) has a drastic effect on the prediction for m_h.

The prospective experimental error on m_h is also shown in figure 2, while no intrinsic theoretical uncertainty from unknown higher-order corrections is included. If this intrinsic uncertainty can be reduced to a level of $\Delta m_h^{\mathrm{intr,future}} \approx 0.1\,\mathrm{GeV}$, its effect in the plot would be roughly as big as the one induced by $\delta m_t = 0.1\,\mathrm{GeV}$. An intrinsic uncertainty of $\Delta m_h^{\mathrm{intr,future}} \approx 1\,\mathrm{GeV}$, on the other hand, would lead to a significant widening of the band of predicted m_h values (similar to the effect of $\delta m_t = 1\,\mathrm{GeV}$). In this case the intrinsic uncertainty would dominate, implying that a reduction of $\delta m_t = 1\,\mathrm{GeV}$ to $\delta m_t = 0.1\,\mathrm{GeV}$ would lead to an only moderate improvement of the overall theoretical uncertainty of m_h.

Confronting the theoretical prediction for m_h with a precise measurement of the Higgs-boson mass constitutes a very sensitive test of the MSSM, which allows us to obtain constraints on the model parameters. The sensitivity of the m_h prediction on M_A shown

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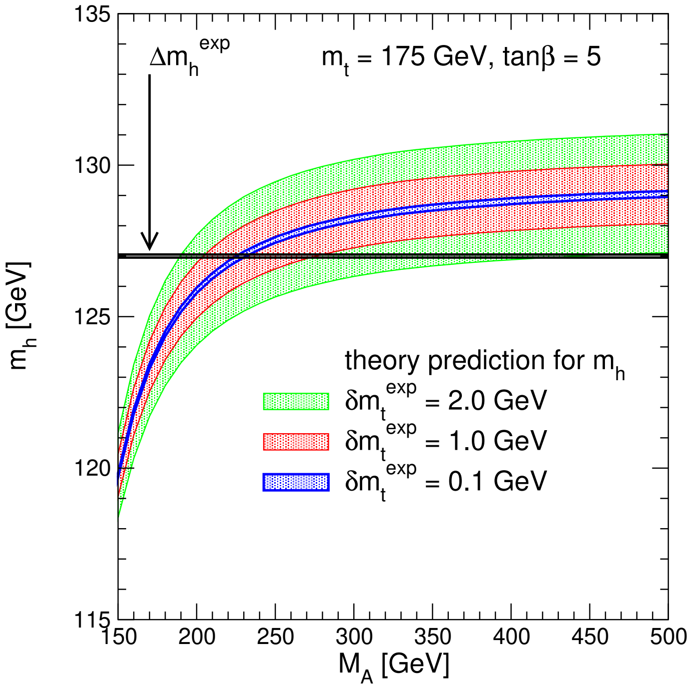

Figure 2: The prediction for m_h in the $m_h^{\max}$ scenario is shown as a function of M_A for $m_t = 175\,\text{GeV}$ and $\tan\beta = 5$. The three bands correspond to $\delta m_t = 1, 2\,\text{GeV}$ (LHC) and $\delta m_t = 0.1\,\text{GeV}$ (LC). The anticipated experimental error on m_h at the LC is also indicated.

in figure 2 cannot directly be translated into a prospective indirect determination of M_A, however, since figure 2 shows the situation in a particular benchmark scenario [46] where, by definition, certain fixed values of all other SUSY parameters are assumed. In a realistic situation the anticipated experimental errors of the other SUSY parameters, and possible effects of intrinsic theoretical uncertainties, have to be taken into account. In the next section, we will analyse the prospects for an indirect determination of SUSY parameters from precision physics in the MSSM Higgs sector. In particular, we will consider two examples of parameter determination in the stop sector of the MSSM.

3.2 Constraints on the parameters of the stop sector

Once a Higgs boson compatible with the MSSM predictions has been discovered, the dependence of m_h on the top and stop sectors can be utilized to determine unknown parameters of the $\tilde{t}$ sector.

The mass matrix relating the interaction eigenstates $\tilde{t}_L$ and $\tilde{t}_R$ to the mass eigenstates $\tilde{t}_1$ and $\tilde{t}_2$ is given by

$$\mathcal{M}^2_{\tilde{t}} = \begin{pmatrix} M^2_{\tilde{Q}_3} + m_t^2 + \cos 2\beta\,(\frac{1}{2} - \frac{2}{3} s_W^2) M_Z^2 & m_t X_t \\ m_t X_t & M^2_{\tilde{U}_3} + m_t^2 + \frac{2}{3}\cos 2\beta\; s_W^2 M_Z^2 \end{pmatrix}, \tag{3.6}$$

where X_t can be decomposed as $X_t = A_t - \mu/\tan\beta$. Here, A_t denotes the trilinear Higgs–$\tilde{t}$ coupling. Assuming that $\tan\beta$ and μ can be determined from other sectors, there are three new parameters in the mass matrix, $M_{\tilde{Q}_3}$, $M_{\tilde{U}_3}$, and A_t. The mass eigenvalues of the stops

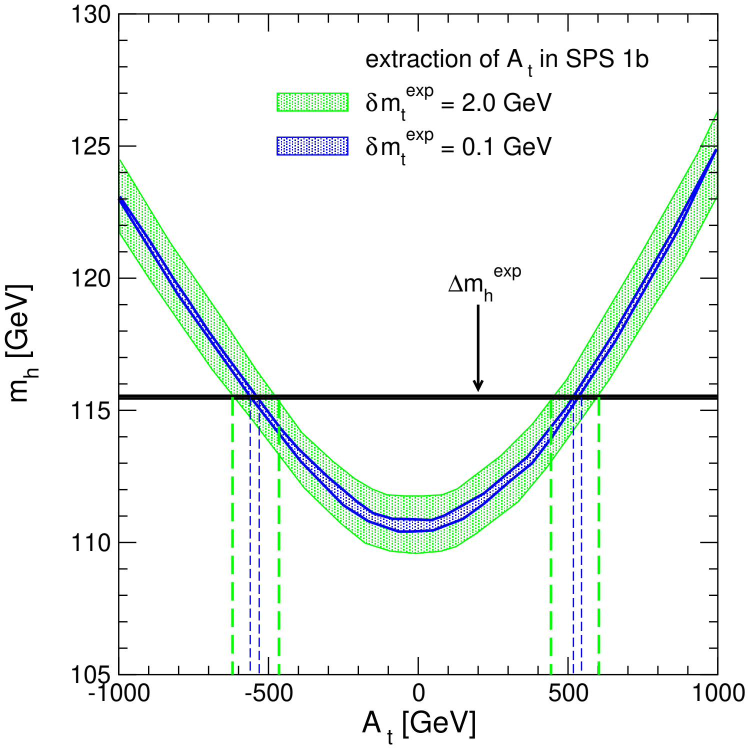

Figure 3: Indirect determination of A_t in the SPS1b scenario for $\delta m_t = 2\,\text{GeV}$ (LHC) and $\delta m_t = 0.1\,\text{GeV}$ (LC). The experimental error on the Higgs boson mass, δm_h^{exp}, is indicated. For the errors on the SUSY parameters we have assumed a 5% uncertainty for $m_{\tilde{t}_1}, m_{\tilde{t}_2}, m_{\tilde{b}_1}, m_{\tilde{b}_2}$ around their values given by SPS1b. M_A is assumed to be determined to about 10%, while for $\tan\beta$ a measurement of $\tan\beta \approx 30 \pm 4.5$ is assumed.

are obtained after a rotation by the angle $\theta_{\tilde{t}}$,

$$\mathcal{M}_{\tilde{t}}^2 \quad \xrightarrow{\theta_{\tilde{t}}} \quad \begin{pmatrix} m_{\tilde{t}_1}^2 & 0 \\ 0 & m_{\tilde{t}_2}^2 \end{pmatrix}. \tag{3.7}$$

A possible future situation would be that the two stop quarks are accessible at the LHC, but are too heavy for direct production at the LC. In this case the masses $m_{\tilde{t}_1}$, $m_{\tilde{t}_2}$ can be determined at the LHC (supplemented with LC input [48, 49]), while only limited information[2] would be obtained on the mixing angle in the stop sector, $\theta_{\tilde{t}}$. Therefore the trilinear coupling A_t could only be loosely constrained. However, the measurement of m_h together with a precise determination of m_t would allow an indirect determination of A_t. This is shown for the benchmark scenario SPS1b [26] in figure 3 (evaluated with *FeynHiggs*). For the errors on the SUSY parameters we have assumed a 5% uncertainty for $m_{\tilde{t}_1}, m_{\tilde{t}_2}, m_{\tilde{b}_1}, m_{\tilde{b}_2}$ around their values given by SPS1b. M_A is assumed to be determined to about 10%, whereas for $\tan\beta$ a measured value of $\tan\beta \approx 30 \pm 4.5$ is assumed. As before, we do not include intrinsic theoretical uncertainties on m_h in the plot. Its effect

[2]For low values of $M_{\tilde{Q}_3}$, $M_{\tilde{U}_3}$ of $\mathcal{O}(200\,\text{GeV})$ a weak sensitivity to $\theta_{\tilde{t}}$ might also be available from the process $pp \to \tilde{t}_1\tilde{t}_1 h$ at the LHC [50].

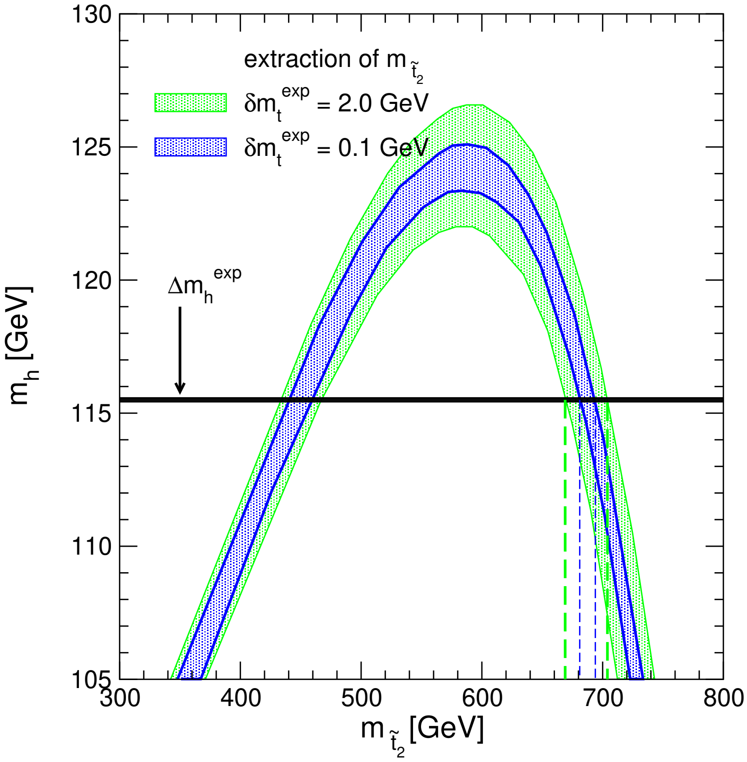

Figure 4: Indirect determination of $m_{\tilde{t}_2}$ from the measurement of m_h for $\delta m_t = 2\,\text{GeV}$ (LHC) and $\delta m_t = 0.1\,\text{GeV}$ (LC). The experimental error on the Higgs boson mass, δm_h^{exp}, is indicated. For the assumed experimental errors of the other SUSY parameters see text.

can most easily be illustrated by replacing the experimental error on m_h by a combination of the experimental error and the intrinsic theoretical uncertainty. An intrinsic theoretical uncertainty in excess of the experimental error on m_h would therefore effectively widen the indicated interval of allowed m_h values.

Figure 3 shows that the experimental error on m_t has a significant impact on the indirect determination of A_t. The LC precision on m_t gives rise to an improvement in the accuracy for A_t by a factor of about 3, to be compared with the case where m_t is known with the accuracy achievable at the LHC.

As another example, in ref. [51] a scenario was analysed where the lighter stop $\tilde{t}_1$ is accessible at a LC with $\sqrt{s} = 500\,\text{GeV}$. In this case, the LC will provide precise measurements of $m_{\tilde{t}_1}$ and $\theta_{\tilde{t}}$. On the other hand, the heavier stop $\tilde{t}_2$ is too heavy to be produced at the LC in this scenario. In such a case, the measurement of m_h can be employed for obtaining indirect limits on $m_{\tilde{t}_2}$. A comparison of this indirect determination of $m_{\tilde{t}_2}$ with a direct measurement at the LHC or a LC at higher energy would provide a stringent consistency test of the MSSM.

In figure 4 we show the allowed region in the $m_{\tilde{t}_2}$–m_h plane, where the following values for the other parameters have been assumed [51, 52]: $m_{\tilde{t}_1} = 180 \pm 1.25\,\text{GeV}$, $\cos\theta_{\tilde{t}} = 0.57 \pm 0.01$, $M_A = 257 \pm 10\,\text{GeV}$, $\mu = 263 \pm 1\,\text{GeV}$, $m_{\tilde{g}} = 496 \pm 10\,\text{GeV}$, $A_b = A_t \pm 30\%$, and a lower bound of 200 GeV has been imposed on the lighter sbottom mass. For $\tan\beta$

only a lower bound of $\tan\beta > 10$ has been assumed, which could for instance be inferred from the gaugino/higgsino sector. For the Higgs-boson mass a measured value of $m_h = 115.5 \pm 0.05\,\text{GeV}$ has been assumed. Concerning the intrinsic theoretical uncertainty of m_h, the same applies as for figure 3 above.

Intersecting the assumed measured value for m_h with the allowed region in the $m_{\tilde{t}_2}$–m_h plane allows an indirect determination of $m_{\tilde{t}_2}$ in this example, yielding $670\,\text{GeV} \lesssim m_{\tilde{t}_2} \lesssim 705\,\text{GeV}$ for $\delta m_t = 2\,\text{GeV}$ (we consider only the intersection at higher values of $m_{\tilde{t}_2}$, since we had assumed that $m_{\tilde{t}_2}$ lies above the LC reach). This precision increases with $\delta m_t = 0.1\,\text{GeV}$ to $680\,\text{GeV} \lesssim m_{\tilde{t}_2} \lesssim 695\,\text{GeV}$, i.e. by a factor of more than 2.

4. Renormalization group running of SUSY-breaking parameters

4.1 Neutralino and chargino masses in mSUGRA

Once SUSY particles are detected and their properties are measured, a major task will be to determine the underlying scheme for supersymmetry breaking out of the experimental data. For this purpose the low-energy parameters are related to high-scale parameters by means of renormalization group equations (RGEs). The precision with which this can be done naturally depends on the amount and precision of the available experimental data. The top quark mass and top Yukawa coupling are important parameters in the renormalization group running and in radiative corrections to SUSY masses. A precise knowledge of m_t will thus be very important for the determination of fundamental-scale SUSY parameters and for testing different models of SUSY breaking.

Let us consider the mSUGRA model [53] as an illustrative example. This model is characterized by five parameters: a common scalar mass m_0, a common fermion mass $m_{1/2}$, and a common trilinear coupling A_0 at the grand unification (GUT) scale, supplemented by $\tan\beta$ and the sign of the higgsino mass parameter μ. The MSSM spectrum is determined from these five parameters by RG evolution. It turns out that the μ parameter is very sensitive to the top quark mass, especially in the case of large m_0. With a top pole mass around $174\,\text{GeV}$, the μ parameter depends only weakly on the value of m_0. Such scenarios are called focus-point scenarios [54]. In these scenarios several large terms cancel, implying that a small change of the top mass can lead to huge effects. Figure 5a shows μ as a function of m_0 for $m_{1/2} = 300\,\text{GeV}$, $A_0 = 0$, $\tan\beta = 10$ and top quark masses of $172--178\,\text{GeV}$. As can be seen, the m_0 dependence of μ (i.e. the absolute value of μ as well as whether or not focus-point behaviour is found) is highly sensitive to the exact value of m_t. Analogously, figure 5b shows μ as a function of m_t for several fixed values of m_0. Again we find a very pronounced dependence on m_t for large m_0. For $m_0 \sim 2\,\text{TeV}$, for instance, a shift in m_t of $1\,\text{GeV}$ leads to a shift in μ of about $100\,\text{GeV}$. In gauge- or anomaly-mediated SUSY breaking, the situation is somewhat more stable, as there are no focus-point-like scenarios. However, an error of $2\,\text{GeV}$ on m_t can still introduce an error of about $50\,\text{GeV}$ for μ. This reduces to an $\mathcal{O}(1\,\text{GeV})$ error for $\delta m_t = 0.1\,\text{GeV}$.

The dependence of μ on m_t directly translates into the predicted chargino and neutralino masses. This is shown in Table 2, where we list the parametric errors on $m_{\tilde{\chi}^0_1}$, $m_{\tilde{\chi}^\pm_1}$ and $m_{\tilde{\chi}^\pm_2}$ for $\delta m_t = 0.1$–$2\,\text{GeV}$ and two values of m_0, 0.5 and 1 TeV. The other parameters

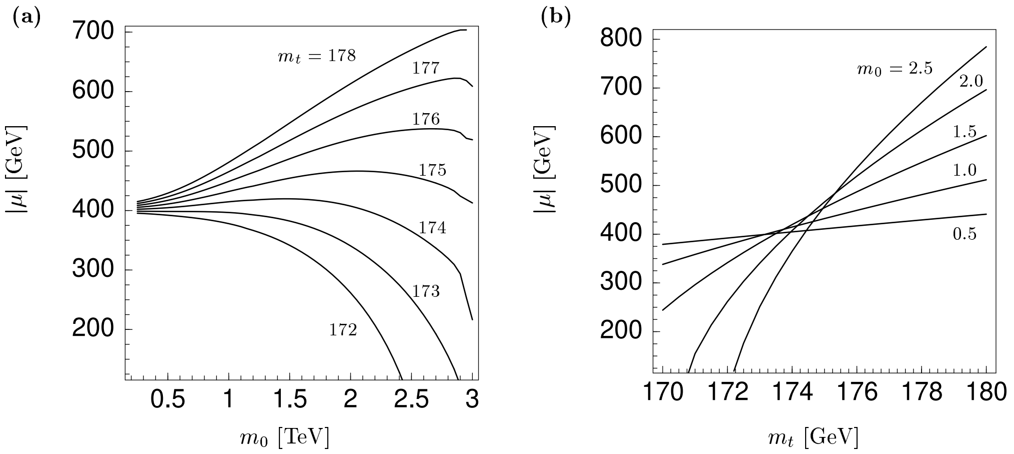

Figure 5: Dependence of μ on (a) m_0 for various values of m_t (in GeV) and (b) m_t for various values of m_0 (in TeV) within the mSUGRA model, for $m_{1/2} = 300\,\text{GeV}$, $A_0 = 0$, $\tan\beta = 10$; calculated with SPHENO 2.0.1 [55].

are $m_{1/2} = 300\,\text{GeV}$, $A_0 = 0$, $\tan\beta = 10$, $\mu > 0$ as above, and $m_t = 175\,\text{GeV}$. The errors on the chargino and neutralino masses scale roughly linearly with the error on the top mass.

As can be seen, a precise knowledge of m_t is essential for precise predictions. Such predictions within a particular model can be used for e.g., exclusion limits or consistency checks. For example, if the properties of $\tilde{\chi}_1^0$, $\tilde{\chi}_2^0$ and $\tilde{\chi}_1^\pm$ are measured with high precision, the parameters M_1, M_2 and μ can be derived without assuming a particular SUSY-breaking scenario [56]. Combining this with information on the squark and gluino masses from the LHC, one may then perform a hypothesis test of mSUGRA (or other models) in a top–down approach, see e.g. [57]. From table 2 it becomes clear that $\mathcal{O}(100\,\text{MeV})$ precision on m_t is necessary for mSUGRA fits in order to match the experimental precision of gaugino mass measurements at a LC.

For completeness we note that at present there is a non-negligible theoretical uncertainty in SUSY mass spectrum calculations from RG running [58]. The main source is the relation between the measured top mass and the Yukawa coupling y_t. The current results have been obtained using the complete SUSY one-loop relation plus the gluonic part of the two-loop relation. Recently also the SUSY-QCD part of the two-loop relation has been given in the literature [59]. In order to fully exploit the LC precision on m_t, further improvements will be necessary, in particular a complete two-loop calculation of y_t.

The relation between the measured top mass and the Yukawa coupling y_t depends also on the precise value of α_s. Shifting the value of α_s by 10^{-3} amounts to shifting $|\mu|$ by 4 GeV for $m_0 = 2.5\,\text{TeV}$. For large $\tan\beta$, the error on the bottom mass has a similar influence. Taking $\tan\beta = 50$ and $m_0 = 2.5\,\text{TeV}$ and the other parameters as above, a shift of 0.2 GeV in the bottom mass induces a shift in $|\mu|$ of 5 GeV. These uncertainties are of the same order of magnitude as those induced by the shift of $\delta m_t = 0.1\,\text{GeV}$. For comparison, at $m_0 = 2.5\,\text{TeV}$ a shift of $\delta m_t = 0.1\,\text{GeV}$ induces a shift in μ of 8 GeV.

δm_t	$m_0 = 0.5\,\text{TeV}$ $\delta m_{\tilde{\chi}_1^0}$	$\delta m_{\tilde{\chi}_1^\pm}$	$\delta m_{\tilde{\chi}_2^\pm}$	$m_0 = 1\,\text{TeV}$ $\delta m_{\tilde{\chi}_1^0}$	$\delta m_{\tilde{\chi}_1^\pm}$	$\delta m_{\tilde{\chi}_2^\pm}$
0.1	0.01	0.06	0.54	0.02	0.15	1.6
0.2	0.02	0.11	1.1	0.05	0.30	3.1
0.5	0.05	0.29	2.7	0.13	0.78	7.8
1.0	0.10	0.59	5.5	0.26	1.7	16.0
2.0	0.21	1.2	11.0	0.59	3.7	32.0

Table 2: Uncertainties in predicted neutralino and chargino masses in mSUGRA (in GeV) due to δm_t for $m_{1/2} = 300\,\text{GeV}$, $A_0 = 0$, $\tan\beta = 10$, $\mu > 0$ and $m_t = 175\,\text{GeV}$; calculated with SPheno2.0.1 [55]. The central values for the masses are $m_{\tilde{\chi}_1^0} = 121.02\,\text{GeV}$, $m_{\tilde{\chi}_1^\pm} = 226.53\,\text{GeV}$, $m_{\tilde{\chi}_2^\pm} = 435.17\,\text{GeV}$ for $m_0 = 500\,\text{GeV}$ and $m_{\tilde{\chi}_1^0} = 123.06\,\text{GeV}$, $m_{\tilde{\chi}_1^\pm} = 230.97\,\text{GeV}$, $m_{\tilde{\chi}_2^\pm} = 456.13\,\text{GeV}$ for $m_0 = 1\,\text{TeV}$. For the other neutralinos, $m_{\tilde{\chi}_2^0} \sim m_{\tilde{\chi}_1^\pm}$, $m_{\tilde{\chi}_{3,4}^0} \sim m_{\tilde{\chi}_2^\pm}$ and $\delta m_{\tilde{\chi}_2^0} \sim \delta m_{\tilde{\chi}_1^\pm}$, $\delta m_{\tilde{\chi}_{3,4}^0} \sim \delta m_{\tilde{\chi}_2^\pm}$.

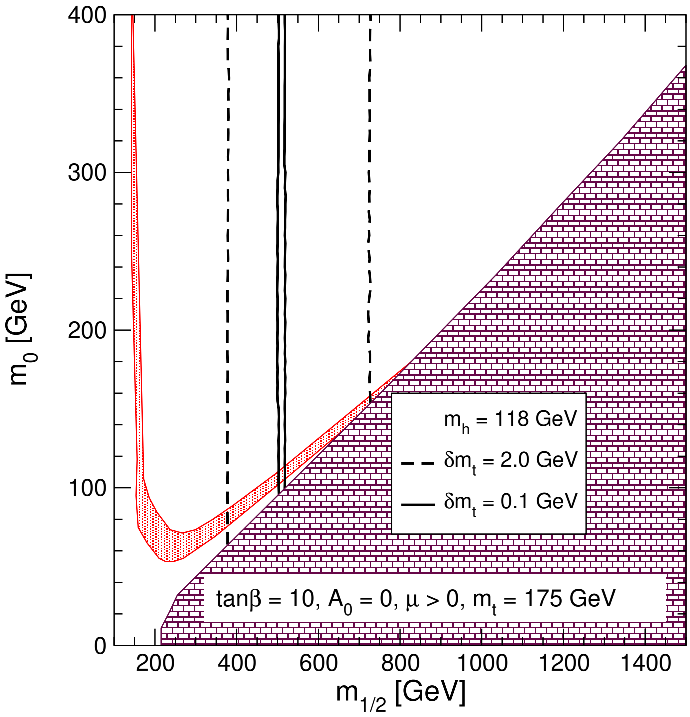

Figure 6: The CDM-allowed region from the WMAP measurement is shown in the mSUGRA scenario in the $m_{1/2}$–m_0 plane for $A_0 = 0$, $\tan\beta = 10$ and $\mu > 0$. The brown (bricked) region is excluded because the LSP is the charged $\tilde{\tau}_1$ in this region. An assumed measurement of $m_h = 118 \pm 0.05\,\text{GeV}$ is shown together with the parametric uncertainty from $\delta m_t = 0.1\,\text{GeV}$ (solid line) and $\delta m_t = 2.0\,\text{GeV}$ (dashed).

4.2 Dark matter constraints in the mSUGRA scenario

Within the mSUGRA scenario, the lightest SUSY particle (LSP), which we take to be the lightest neutralino, is a good candidate for cold dark matter (CDM). After the recent WMAP measurements [60], the required amount of CDM given by neutralinos is narrowed

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down to $0.094 \leq \Omega_\chi h^2 \leq 0.129$. In figure 6 we show the CDM-allowed region in the $m_{1/2}$–m_0 plane for $A_0 = 0$, $\tan\beta = 10$ and $\mu > 0$ [61, 62]. The evaluation of the CDM-allowed region has been made for $m_t = 175\,\mathrm{GeV}$; however, no relevant change is expected from variations in m_t by $\pm 2\,\mathrm{GeV}$. Also for large $\tan\beta$, the CDM-allowed region is not significantly changed by variations in m_t for the parameter space shown in figure 6. If the Higgs boson mass is measured, it can be used to further constrain the mSUGRA parameter space. In figure 6 we have assumed an experimental determination of $m_h = 118 \pm 0.05\,\mathrm{GeV}$. The parametric uncertainty induced by δm_t is indicated. For $\delta m_t = 2\,\mathrm{GeV}$ the allowed regions are $400\,\mathrm{GeV} \lesssim m_{1/2} \lesssim 750\,\mathrm{GeV}$ and $80\,\mathrm{GeV} \lesssim m_0 \lesssim 160\,\mathrm{GeV}$. These uncertainties shrink to $500\,\mathrm{GeV} \lesssim m_{1/2} \lesssim 520\,\mathrm{GeV}$ and $100\,\mathrm{GeV} \lesssim m_0 \lesssim 120\,\mathrm{GeV}$ with $\delta m_t = 0.1\,\mathrm{GeV}$. This corresponds to an improvement by factors of 17 and 4 for $m_{1/2}$ and m_0, respectively.

4.3 Bottom-up determination of SUSY-breaking parameters

If LHC measurements of the MSSM spectrum are complemented by high precision measurements at a prospective LC with sufficiently high energy, one can try to reconstruct the original theory at the high scale in a model-independent way; see ref. [63, 64]. In [63, 64] it has been assumed that m_t is known with $\delta m_t = 0.1\,\mathrm{GeV}$ accuracy. In the following we will study the situation if m_t is known less precisely.

We shortly summarize the used procedure, as further details can be found in [63]–[55]. We take the masses and cross sections of a particular point in the SUSY parameter space together with their expected experimental errors from the LHC and a $\sqrt{s} = 800\,\mathrm{GeV}$ LC. We assume that electrons can be polarized to 80% and positrons to 40% at the LC [65]. We then fit the underlying SUSY-breaking parameters (at the electroweak symmetry breaking scale, $Q_{\mathrm{EWSB}} = \sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}}$) to these observables. An initial set of parameters is obtained by inverting the tree-level formulas for masses and cross sections. This set serves as a starting point for the fit, which is carried out with MINUIT [66] to obtain the complete correlation matrix. In the fit, the complete spectrum is calculated at the one-loop level using the formulas given in [67]. In the case of the neutral Higgs bosons as well as of the μ parameter, the two-loop corrections as given in [34, 35, 68] are included. In addition, the cross sections for third-generation sfermions at a LC are calculated (including the effect of initial-state radiation [69] and, in the case of squarks, also SUSY-QCD corrections [69, 70]). The low-scale SUSY-breaking parameters and the errors on them are then run up to the high-energy (GUT) scale. In this way one can check the extent to which the original theory can be reconstructed. The new ingredient with respect to refs. [63, 64] is that we include here the effect of the uncertainty of the top mass and all trilinear couplings. The effects of the m_t error are: (i) the top Yukawa coupling gets changed and this affects the RGE running of several other parameters, such as y_b, A_t, $M^2_{\tilde{Q}_3}$, $M^2_{\tilde{U}_3}$ and $M^2_{H_2}$ (the mass parameter of the second Higgs doublet); (ii) m_t enters directly the calculation of the stop masses; (iii) m_t enters the loop-corrected relations between third-generation squark masses, chargino masses, neutralino masses, the gluino mass, the Higgs masses and the underlying parameters in the lagrangian. For m_h, we take $\Delta m_h = \sqrt{(\delta m_h^{\mathrm{exp}})^2 + (\Delta m_h^{\mathrm{intr}})^2}$, with $\delta m_h^{\mathrm{exp}} = 50\,\mathrm{MeV}$, and we use here a future intrinsic uncertainty of $\Delta m_h^{\mathrm{intr}} = 0.5\,\mathrm{GeV}$.

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	$\delta m_t = 0.1\,\mathrm{GeV}$		$\delta m_t = 1\,\mathrm{GeV}$		$\delta m_t = 2\,\mathrm{GeV}$	
	Q_{EWSB}	M_{GUT}	Q_{EWSB}	M_{GUT}	Q_{EWSB}	M_{GUT}
A'_t	-448 ± 17	-56 ± 25	-445 ± 23	-57 ± 38	-445 ± 34	-56 ± 54
$M^2_{\tilde{U}_3}$	186 ± 10	44^{+30}_{-26}	189 ± 12	52^{+47}_{-30}	189 ± 15	54^{+64}_{-46}
$M^2_{\tilde{Q}_3}$	267 ± 7	42^{+20}_{-16}	268 ± 8	45^{+25}_{-16}	268 ± 12	45^{+30}_{-20}
$M^2_{H_2}$	-127.5 ± 0.3	42^{+34}_{-20}	-127.7 ± 0.3	53^{+38}_{-27}	-127.5 ± 0.4	53^{+51}_{-39}

Table 3: Absolute errors on the parameters A'_t (in [GeV]), $M^2_{\tilde{U}_3}$, $M^2_{\tilde{Q}_3}$, and $M^2_{H_2}$ at Q_{EWSB} and M_{GUT} (in $[10^3\,\mathrm{GeV}^2]$) for different values of δm_t. The mSUGRA point probed is defined by eq. (4.1). The 'true' values of the parameters at the GUT scale are $A'_t = 51\,\mathrm{GeV}$, $M^2_{\tilde{U}_3} = M^2_{\tilde{Q}_3} = M^2_{H_2} = 40 \times 10^3\,\mathrm{GeV}^2$.

The SUSY-breaking parameters that are most sensitive to m_t are A'_t, $M^2_{\tilde{Q}_3}$, $M^2_{\tilde{U}_3}$ and the Higgs mass parameter $M^2_{H_2}$. Note that we take $A'_t \equiv A_t \cdot y_t$, where y_t is the top Yukawa coupling, as input for the fit, because it is this parameter that appears in the lagrangian.

As an example we take the mSUGRA parameters

$$m_0 = 200\,\mathrm{GeV}\,, \quad m_{1/2} = 250\,\mathrm{GeV}\,, \quad A_0 = -100\,\mathrm{GeV}\,, \quad \tan\beta = 10\,, \quad \mathrm{sign}(\mu) = (+) \tag{4.1}$$

to generate the SUSY spectrum — then 'forgetting' this origin of the masses and cross sections. The value of A'_t at the GUT scale is in this example $-51\,\mathrm{GeV}$. The same experimental errors as given in [64] are assumed. This holds except for m_h as discussed above, and we also assume more conservatively that the trilinear couplings in the $\tilde{b}$ and $\tilde{\tau}$ sector A_b and A_τ can be determined within 30%. (See ref. [71] for first attempts in this direction which indicate that this might be achievable.) This assumption does not influence the errors on the parameters A'_t, $M^2_{\tilde{Q}_3}$ and $M^2_{\tilde{U}_3}$ and $M^2_{H_2}$ at the electroweak scale, but has some impact on their RG evolution in the case of $\delta m_t = 0.1\,\mathrm{GeV}$: doubling the error on A_b roughly increases the errors on their corresponding GUT values by about 30%. The impact of A_b is less pronounced for larger δm_t because the error on the top mass then dominates.

Table 3 lists the central values and the 1σ errors of the parameters A'_t, $M^2_{\tilde{Q}_3}$, $M^2_{\tilde{U}_3}$ and $M^2_{H_2}$ at the electroweak scale and the GUT scale. The errors both at Q_{EWSB} and at M_{GUT} clearly depend on δm_t. The other parameters are less affected by m_t. The errors shown in table 3 are to some extent correlated, because the stop mass parameters are not only constrained by the stop masses and the stop cross sections, but also by m_h through the higher order corrections (see also section 3.2). The asymmetry of the errors of the scalar mass parameters squared at the GUT scale is due to the fact that $(A'_t)^2$ enters the corresponding RGEs.

In figure 7 we show the evolution of the parameters discussed above for the three top quark mass errors, $\delta m_t = 0.1\,\mathrm{GeV}$ (blue/dark shaded), $\delta m_t = 1\,\mathrm{GeV}$ (green/medium) and $\delta m_t = 2\,\mathrm{GeV}$ (yellow/light). It is apparent that the error on the top mass affects all four parameters. The 'asymmetric' increase of the errors is due to the different central values of the parameters obtained in the different fits (see also Table 3) and is also a consequence of the correlations between the different errors at the electroweak scale.

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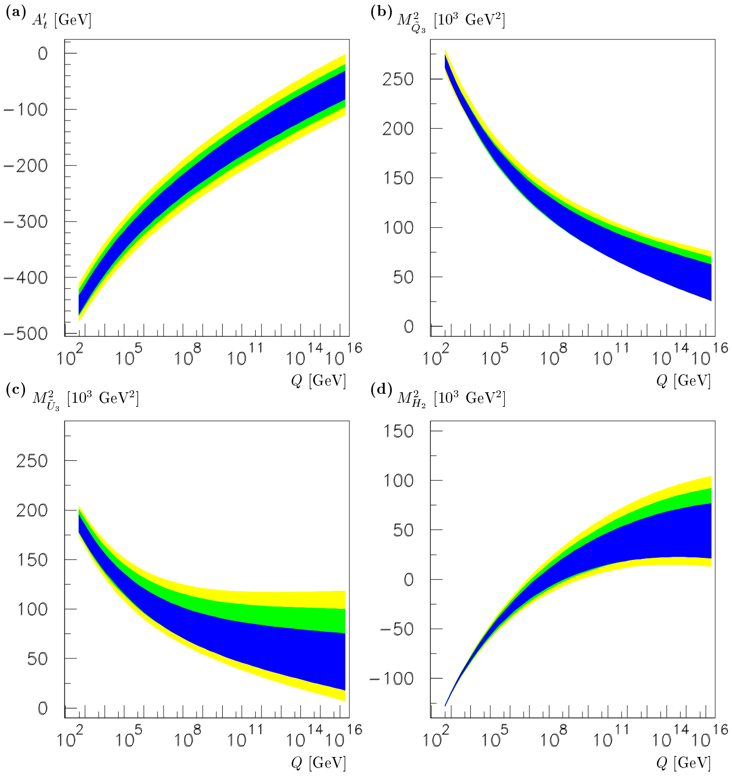

Figure 7: Comparison of RGE running of (*a*) A'_t, (*b*) $M^2_{\tilde{Q}_3}$, (*c*) $M^2_{\tilde{U}_3}$ and (*d*) $M^2_{H_2}$, for the three top quark mass errors $\delta m_t = 0.1\,\text{GeV}$ (blue/dark shaded), $\delta m_t = 1\,\text{GeV}$ (green/medium) and $\delta m_t = 2\,\text{GeV}$ (yellow/light). The mSUGRA scenario is defined by eq. (4.1). The 'true' values of the parameters at the GUT scale are $A'_t = 51\,\text{GeV}$, $M^2_{\tilde{U}_3} = M^2_{\tilde{Q}_3} = M^2_{H_2} = 40 \times 10^3\,\text{GeV}^2$. The widths of the bands indicate the 1σ CL.

One clearly sees that δm_t should be about 0.1 GeV in order to yield the $\tilde{t}$ sector parameters at the high scale with a reasonable precision. The errors roughly increase by 60% (100%) if the error of m_t is increased from 0.1 GeV to 1 GeV (2 GeV). Moreover, the smaller the error of m_t, the closer the central values of $M^2_{\tilde{U}_3}$, $M^2_{\tilde{Q}_3}$ and $M^2_{H_2}$ to their 'true' values. This is in particular important in view of models where unification of the scalar masses is imposed at the Planck scale instead of the GUT scale: these models predict that at M_{GUT} the third-generation mass parameters as well as the Higgs mass parameters are different from sfermion mass parameters of the first two generations [72].

Let us finally comment on the influence of the errors on α_s and m_b. Shifting α_s by 10^{-3} leads to a per-mille error on the high scale parameters. When $\tan\beta$ is small, an error of 0.2 GeV on m_b induces an error on the discussed high scale parameters, which is well below a per mille. For large $\tan\beta$, the situation is more complicated because then also the errors on A_b, $M^2_{\tilde{D}_3}$ (the soft SUSY-breaking parameter in the $\tilde{b}$ sector) and $M^2_{H_1}$ (the Higgs mass parameter of the first doublet) play a role and the errors on the stop mass parameters at the high scale can easily be increased by several per cent. Concerning the intrinsic theoretical uncertainties from unknown higher-order corrections, the situation is as discussed in section 4.1.

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5. Conclusions

In this paper we have analysed the physics impact of the precise experimental determination of the top quark mass at a prospective e^+e^- Linear Collider down to $\delta m_t \lesssim 100\,\mathrm{MeV}$ with respect to the envisaged LHC precision of $\delta m_t \approx 1$–$2\,\mathrm{GeV}$.

Within the SM and the MSSM, a precise knowledge of the top quark mass has a strong impact on the prediction of electroweak precision observables such as M_W and $\sin^2\theta_{\mathrm{eff}}$, which receive higher-order corrections $\sim m_t^2$. Stringent internal consistency checks of both models are only possible with the LC accuracy on m_t. In particular, a precision of m_t significantly better than 1 GeV will be necessary in order to exploit the prospective precision of the EWPO. The precise value of m_t furthermore improves the indirect determination of the top Yukawa coupling by a factor of about 2.

The precision of the top quark mass is particularly important for the MSSM Higgs sector, since the parametric error of m_h, being $\sim m_t^4$, is roughly given by $\Delta m_h^{\mathrm{para}} \approx \delta m_t$. We have demonstrated that precision physics in the MSSM Higgs sector will require a very precise knowledge of m_t. The accuracy of $\delta m_t = 0.1\,\mathrm{GeV}$ can, however, only be fully exploited provided that the intrinsic theoretical uncertainty in m_h can be reduced to a similar level. We have also analysed indirect constraints on the parameters of the stop sector, taking into account anticipated future experimental uncertainties. In the examples we have investigated, the LC accuracy on m_t gives rise to an improvement by a factor of 2–3 in the indirect determination of A_t or $m_{\tilde{t}_2}$.

The renormalization group running of SUSY-breaking parameters is also very sensitive to m_t. Within the mSUGRA scenario the error on the predicted neutralino and chargino masses directly scales with δm_t, leading to a factor of 10 improvement when going from hadron collider to LC precision in m_t. In focus-point scenarios the dependence on m_t is even more pronounced. When the relic-density constraint is combined with a prospective Higgs mass measurement, the improved precision on m_t leads to a significant reduction of the allowed mSUGRA parameter space. In the bottom–up approach for reconstructing the high-energy theory, a precise knowledge of m_t improves the uncertainty of the high-scale parameters by a factor of about 2.

Acknowledgments

We thank D.J. Miller for suggesting this project. We are grateful to J. Erler and M. Winter for helpful discussions. We thank K. Olive for providing data for the CDM evaluation. This work has been supported by the European Community's Human Potential Programme under contract HPRN-CT-2000-00149 'Physics at Colliders'. W.P. is supported by the 'Erwin Schrödinger fellowship No. J2272' of the 'Fonds zur Förderung der wissenschaftlichen Forschung' of Austria and partly by the Swiss 'Nationalfonds'.

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